Solutions - Midterm Exam

(October 17th @ 5:30 pm)

Presentation and clarity are very important! Show your procedure!

PROBLEM 1 (24 PTS)

a) Complete the following table. The decimal numbers are unsigned: (5 pts.)

Decimal	BCD	Binary	Reflective Gray Code
27	00100111	11011	10110
57	01010111	111001	100101
133	000100110011	10000101	11000111

b) Complete the following table. The decimal numbers are signed. Use the fewest number of bits in each case: (15 pts.)

	REPRESENTATION		
Decimal	**Sign-and-magnitude**	**1's complement**	**2's complement**
-1	11	10	1111
-7	1111	1000	1001
11	01011	01011	01011
-27	111011	100100	100101
0	00	1111	0
-64	11000000	10111111	1000000

c) Convert the following decimal numbers to their 2's complement representations. (4 pts)

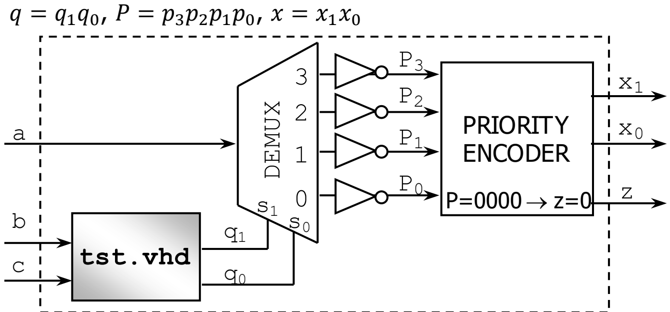
✓ -27.25
+27.25 = 011011.01 ⇒ -27.25 = 100100.11

✓ 26.5
26.5 = 011010.1

PROBLEM 2 (17 PTS)

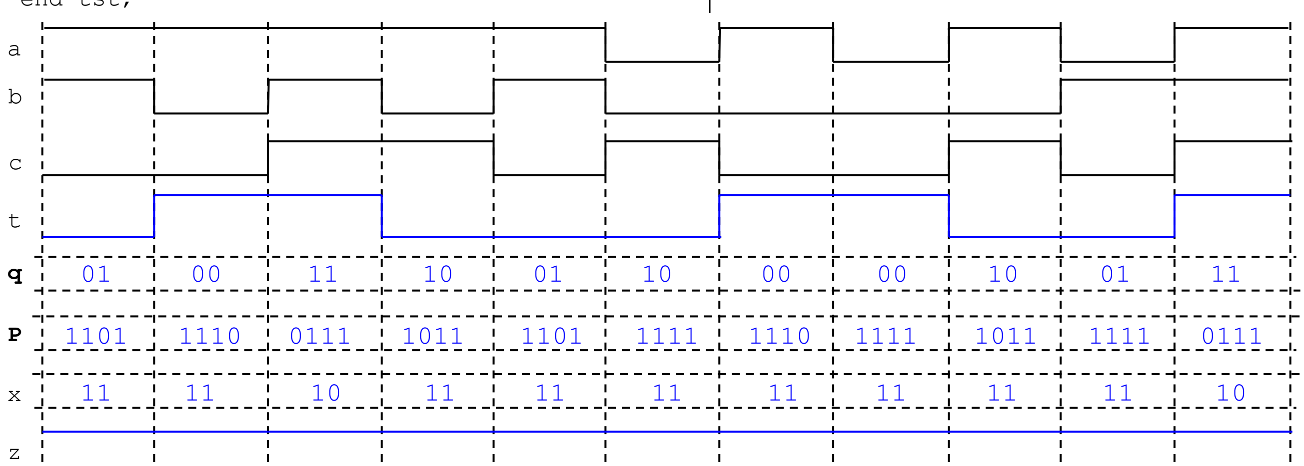
- Complete the timing diagram of the following circuit. The VHDL code (tst.vhd) corresponds to the shaded circuit.

$q = q_1q_0$, $P = p_3p_2p_1p_0$, $x = x_1x_0$



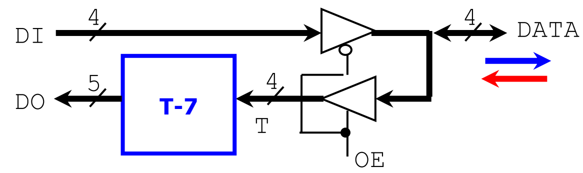

```
library ieee;
use ieee.std_logic_1164.all;

entity tst is
  port (b, c : in std_logic;
        q: out std_logic_vector(1 downto 0));
end tst;
```

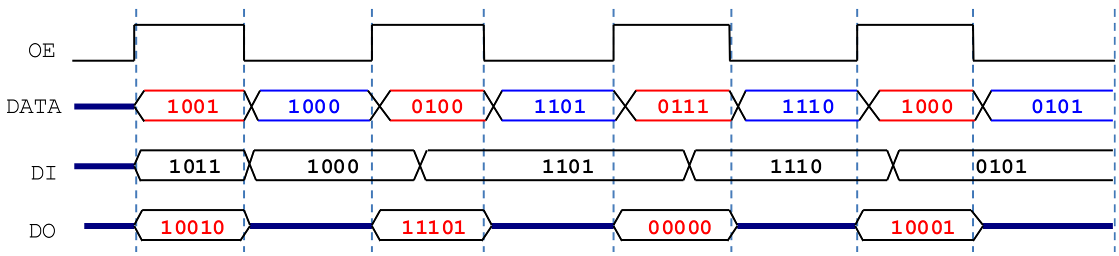
```
architecture bhv of tst is

   signal t: std_logic;

begin

  t <= b xnor c;
  process (b,c,t)
  begin
     q <= b & c;
     if t = '0' then
        q <= c & b;
     end if;
  end process;

end bhv;
```



PROBLEM 3 (12 PTS)

- Complete the timing diagram (signals *DO* and *DATA*) of the following circuit. The circuit in the blue box computes the signed operation T-7, with the result having 5 bits. T is a 4-bit signed (2C) number.





PROBLEM 4 (12 PTS)

- The figure below depicts the entire memory space of a microprocessor. Each memory address occupies one byte. 1KB = 2^{10} bytes, 1MB = 2^{20} bytes, 1GB = 2^{30} bytes
  - ✓ What is the size (in bytes, KB, or MB) of the memory space? What is the address bus size of the microprocessor? (3 pts.)

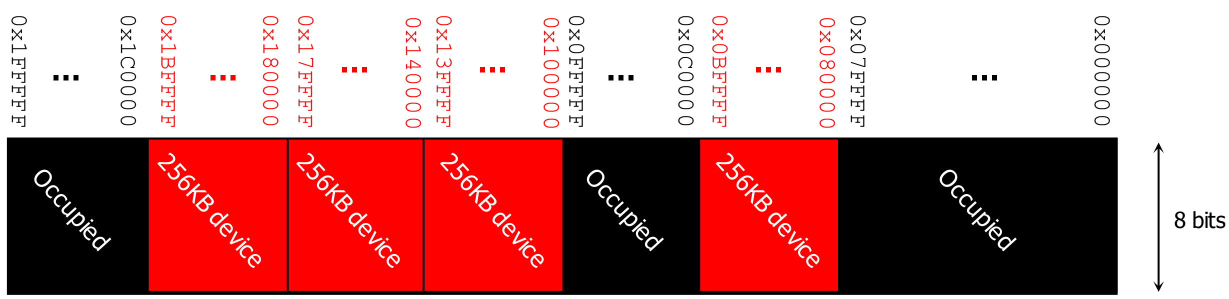
    Address Space: `0x000000` to `0x1FFFFF`. To represent all these addresses, we require 21 bits. So, the address bus size of the microprocessor is 21 bits. The size of the memory space is then 2^{21} = 2 MB.

  - ✓ If we have a memory chip of 256 KB, how many bits do we require to address those 256 KB of memory? (1 pt.)

    256 KB memory device: 256KB = 2^{18} bits. Thus, we require 18 bits to address the memory device.

  - ✓ We want to connect the 256 KB memory chip to the microprocessor. For optimal implementation, we must place those 256 KB in an address range where every address shares some MSBs. Provide a list of all the possible address ranges that the 256 KB memory chip can occupy. You can only use the non-occupied portions of the memory space as shown below.

    - `0x080000` to `0x0BFFFF`
    - `0x100000` to `0x13FFFF`
    - `0x140000` to `0x17FFFF`
    - `0x180000` to `0x1BFFFF`



PROBLEM 5 (18 PTS)

a) Perform the binary unsigned subtraction of these unsigned integers. Use the fewest number of bits n to represent both operators. Indicate every borrow from b_0 to b_n. Determine whether we need to keep borrowing from a higher byte. (6 pts)

✓ `30 - 47`

Borrow out! → $b_6=1$ $b_5=0$ $b_4=1$ $b_3=1$ $b_2=1$ $b_1=1$ $b_0=0$

```
30 = 0x1E =  0 1 1 1 1 0 -
47 = 0x2F =  1 0 1 1 1 1
-------------------------
             1 0 1 1 1 1
```

b) Perform the binary operation of these numbers, where numbers are represented in 2's complement. Indicate every carry from c_0 to c_n. Use the fewest number of bits to represent the summands and the result so that overflow is avoided. (8 pts)

✓ 30 − 47

n = 7 bits

$c_7=0$ $c_6=0$ $c_5=1$ $c_4=0$ $c_3=0$ $c_2=0$ $c_1=0$ $c_0=0$

$c_7 \oplus c_6=0$
No Overflow

```
 30 = 0 0 1 1 1 1 0 +
-47 = 1 0 1 0 0 0 1
--------------------
-17 = 1 1 0 1 1 1 1
```

30 − 47 = −17 ∈ $[-2^6, 2^6-1]$ → no overflow

c) Perform binary multiplication of the following numbers that are represented in 2's complement arithmetic. (4 pts)

✓ -9 x 12

```
1 0 1 1 1 x        1 0 0 1 x
0 1 1 0 0          1 1 0 0
                 ---------
                   0 0 0 0
                 0 0 0 0
               1 0 0 1
             1 0 0 1
           ---------------
           0 1 1 0 1 1 0 0

           1 0 0 1 0 1 0 0
```

PROBLEM 6 (17 PTS)

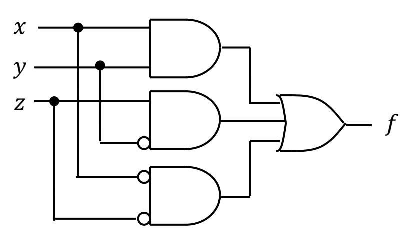
- Given the following Boolean function: $f(x,y,z) = \prod M(3,4)$

a) Provide the simplified expression for f and sketch this circuit using logic gates. (4 pts)

x	y	z	f
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

z \ xy	00	01	11	10
0	1	1	1	0
1	1	0	1	1

$f = xy + \bar{y}z + \bar{x}\bar{z}$



b) Implement the previous circuit using <u>ONLY</u> 2-to-1 MUXs (AND, OR, NOT, XOR gates are not allowed). (13 pts)

$f(x,y,z) = \bar{x}f(0,y,z) + xf(1,y,z) = \bar{x}(\bar{y}z + \bar{z}) + x(y + \bar{y}z) = \bar{x}g(y,z) + xh(y,z)$
$g(y,z) = \bar{y}g(0,z) + yg(1,z) = \bar{y}(1) + y(\bar{z})$
$h(y,z) = \bar{y}h(0,z) + yh(1,z) = \bar{y}(z) + y(1)$

Also: $\bar{z} = \bar{z}(1) + z(0)$